The Laws of Probability

Probability numbers are combined using their own logical arithmetic. The rules by which we manipulate probabilities are called the laws of probability. These laws are helpful to us in the management of risk. Once determined, we can manipulate the probabilities of events to assess proper control or financing alternatives. We can also use the laws to determine the probability of unknown events.

Perhaps the simplest of these laws defines the relationship between a probability and its complement. That is, given the probability that x occurs (p(x)) what is the probability it does not occur? As the probability of all possible outcomes must add up to 1 (certainty), the probability x does not occur, frequently noted as p(x), is 1 minus p(x). Another important combination is the probability of a conjunction of multiple events. For example, what is the probability that if I toss two identical coins both will land "heads"? Or, equivalently, what is the probability that in two consecutive tosses of the same coin it will land "heads"? If we note p(x) as the probability of the first event and p(y) as the probability of the second event that event x and y will occur is calculated as p(x) times p(y). We call this the conjunctive probability of x and y. In our coin example, the probability a fair coin will land "heads" on the first toss and the second is 0.5 x 0.5 = 0.25.

What if we are interested in a coin landing heads on either of the two tosses? That is, we are curious about the occurrence of event x or y. In the management of risk, we may be interested, for example, in the probability that we will have either a major property loss or a major products liability loss this year. The probability in this case is for disjunctive events. The formula for this combination is p(x) plus p(y), minus p(x) times p(y). We subtract the term "p(x) times p(y)" in order to exclude the conjunctive probability that the two events occur together. The probability of a coin landing "heads" on the first toss or the second is therefore (0.5+0.5)–(0.5x0.5), or 0.75.

Many other laws for the combination and manipulation of probabilities exist. Most are intuitive, although they can get complicated. The simple laws shown above along with a few variations permit us to deal with many types of probabilistic phenomena in a reasonable fashion.

The fact that effective real-world decision makers eschew numerical probability representations does not mean that the notion of probability is not part of decision making. A major problem with purely mathematical representations is that while numerical probabilities of events certainly exist, they may be difficult (if not impossible) to determine exactly. This is especially true in complex and dynamic environments where information is scarce. Ignoring probabilities is not the answer. A little knowledge is better than none, and even a little knowledge can be very useful when dealing with risk. To the extent we can *approximate* the probability of events of interest, we should. We will discuss the problem of imperfect knowledge of probabilities later on.